

# Correlation function in Field - Feynman hadronization model

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Very successful hadronization model for the production of mesons in the jets of initial quarks was proposed thirty years ago by Field and Feynman. The model is used so far in the design of experiments and for comparison with the experimental data. In this work it is shown that correlation function in that model, as a function of the energies of two final hadrons has interesting properties (energies are taken in fractions of initial quark energy, and equal  $z_1$  and  $z_2$ , respectively). In particular, for some pairs of mesons there are kinematic regions over variable  $z_2$  (at fixed values of  $z_1$ ), where it is positive and unexpectedly large. The experimental investigation of the correlation functions in these kinematic regions has twofold interest, from one side it can help to investigate some delicate properties of quark jets and from the other side can help to verify the physical suppositions of model, and to obtain more precise values of parameters.

## I. INTRODUCTION

In the paper [1] (hereafter called FF1), the authors investigated the consequences of the assumption that the high-transverse-momentum particles seen in hadron-hadron collisions are produced by a single, hard, large-angle elastic scattering of quarks, one from the target and one from the beam. The fast outgoing quarks were assumed to fragment into a cascade jet of hadrons. The distribution of quarks in the incoming hadrons were determined from lepton-hadron inelastic scattering data, together with certain theoretical constraints such as sum rules, etc. The manner in which quarks cascade into hadrons was determined from particle distributions seen in lepton-hadron and lepton-lepton collisions supplemented by theoretical arguments. Parameterizations for Single-hadron Fragmentation Functions (SFF) were obtained. In the next work [2] a new, much simpler, parameterization for these functions was provided. The answers to many other questions concerning the details of the hadronization process were given. This hadronization model for the production of mesons in the jets of initial quarks and antiquarks turned out very successful and is used so far in the design of experiments in which quark jets may be observed and for comparison with the available experimental data. It provides a parameterization of the properties of the jet of mesons generated by a fast outgoing quark. It is assumed that the meson that contains the original quark leaves momentum and flavor to a remaining jet in which the particles are distributed (except for scaling of the energy and possible changes of flavor) like those of the original jet. One function, the probability  $f(\eta)$  that the remaining jet has a fraction  $\eta$  of the momentum of the original jet, is chosen (as a parabola) so the final distribution of charged hadrons agrees with data from lepton experiments. All the properties of quark jets are determined from  $f(\eta)$  and three parameters: a) the degree that SU(3) is broken in the formation of new quark-antiquark pairs ( $s\bar{s}$  is taken as half as likely as  $u\bar{u}$ )  $u\bar{u} : d\bar{d} : s\bar{s} = \gamma_u : \gamma_d : \gamma_s = 1 : 1 : \frac{1}{2}$  (as usually, it is adopted the well-known condition follow-

ing from the isospin symmetry  $\gamma_u = \gamma_d$ ) and for the sum of probabilities it is taken  $\gamma_u + \gamma_d + \gamma_s = 1$ , because the probabilities of the production of the heavier quarks are neglected; b) the spin nature of the primary mesons (in the basic version of model is assumed to be vector and pseudoscalar with equal probability)  $\alpha_v = \alpha_{ps} = \frac{1}{2}$ ; c) the mean transverse momentum given to these primary mesons<sup>1</sup>. In the rest of the paper we will frequently refer to the Ref. [2] and for this reason it is convenient to denote its as FF2, and model, which was developed there to denote as Field-Feynman Quark Jet Model (FFQJM).

We would like present here the explanation of the success of the FFQJM, which was given by Gottschalk in his lectures [3]: "Specifically, the FFQJM is not a "theory" of how hadronization works. Rather, it is simply a parameterization of hadron formation in jets which reproduces two important experimental facts: 1) approximate scaling of the energy distributions of hadrons within a jet; 2) limited transverse momenta of the produced hadrons with respect to the jet axis. These qualitative features of hard scattering data are not "explained" or "derived" in the FFQJM. Rather, the FFQJM is a parameterization of hadron production from fast quarks which is constructed in such a manner that the above experimental observations are reproduced."

Let us briefly discuss the parameters from points a) and b) of FFQJM presented above. They play important role in the understanding of the hadronization process. In the early model of Field and Feynman presented in FF1, the point a) coincides with that from FFQJM, but point b) differs. For its in FF1 is chosen simple version  $\alpha_{ps} = 1$ ,  $\alpha_v = 0$ . In the simple Lund model [4], which has very large similarities with the FFQJM, the point a) coincides with latter, but for parameters of point b) the ratio is chosen in the form  $\alpha_v : \alpha_{ps} = 3 : 1$  as given by statistical spin counting. In the later version of Lund model, the so called standard Lund model [5] the point

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<sup>1</sup> In this work we do not consider the transverse momenta of hadrons and suppose that integration over them are performed.

a) is changed to  $\gamma_u : \gamma_d : \gamma_s = 1 : 1 : \frac{1}{3}$  which is followed from the tunneling production mechanism, but point b) coincides with one from FFQJM, because authors state that the spin-spin forces enhance the production of the lighter pseudoscalars, from the naive 3 : 1 vector to pseudoscalar ratio to something closer to 1 : 1. Later these parameters were investigated in detail in the process of  $e^+e^-$  annihilation (see, for instance, Refs. [6] and [7]). In [7] it is shown that the production rates of light-flavor hadrons are set by their masses, spins, strangeness suppression, and strongly influenced by the spin-spin interactions of their quarks. The vector-to-pseudoscalar meson and decuplet-to-octet baryon suppresions are the same and are explained by the hyperfine mass splitting. The strangeness suppression factor,  $\lambda = 2\gamma_s/(\gamma_u + \gamma_d) = 0.295 \pm 0.006$ , is the same for mesons and baryons. It is related to the difference in the constituent quark masses,  $\lambda = e^{-(m_s - m_l)/T}$ , where  $m_l = m_u = m_d$ , at the temperature,  $T = 142.4 \pm 1.8$  MeV. Comparison of the parameters from FFQJM with the ones obtained in other models shows that their values differ significantly. This means, that values of parameters in FFQJM were determined with large uncertainties. Inclusion in consideration the new sets of experimental data and new observables can improve description and shift the parameters to their precise values.

In FFQJM, using formalism based on the recursive principle and available experimental data, were obtained SFF  $D_q^h(z)$ , which are the distributions of hadrons of flavor <sup>2</sup>  $h$  from a quark  $q$ , which carry away the fraction  $z$  of its energy, and Dihadron Fragmentation Functions (DFF)  $D_q^{h_1 h_2}(z_1, z_2)$ , which are the distributions of hadrons of flavors  $h_1$  and  $h_2$  from a quark  $q$ , which carry away the fractions  $z_1$  and  $z_2$  of its energy, respectively. Using these results, authors proposed several experiments, which can verify model. Among others, they proposed to measure the correlation functions<sup>3</sup> for oppositely and identically charged hadrons. It was predicted, that correlation function for the oppositely charged hadrons must show the characteristic short-range correlation behavior. The correlation between two identically charged hadrons, on the other hand, was predicted to be quite small. In FF2 the correlation function of two hadrons was presented as a function of their "z rapidity" given by  $Y_z = -\ln(z)$ . Were considered energies large enough that a plateau was developed. Rapidity of the first hadron was fixed in region of plateau,  $Y_{z_1} = 4.0$  (which corresponds  $z_1 \approx 0.02$ ) and  $Y_{z_2}$  was changed in wide region  $0 < Y_{z_2} < 6$ . As it is well known, in this model energy and momentum of the quark jets

do not precisely conserved. The violation of the energy conservation reaches a few per cents. In these conditions the consideration of hadrons with small enough energy may be doubtful.

In this work we study the correlation function in the framework of FFQJM in detail, for the one definite reaction, the semi-inclusive electroproduction of hadrons on proton in Deep-Inelastic Scattering (DIS) region, at moderate energies (energy of the virtual photon or, which is the same, the energy of the initial quark in order of 10-20 GeV), where the plateau does not formed and where the small values of  $z$  in order of 0.02 do not belong to the current fragmentation region. Instead of pairs of oppositely and identically charged hadrons we consider pairs of hadrons having definite charge and flavor; and instead of rapidity, which has very narrow changing region at moderate energies, we use variable  $z$ . It is shown that the correlation function has interesting properties. In particular for some pairs of hadrons there are kinematic regions over variable  $z_2$  (at fixed values of  $z_1$ ) where it is positive and unexpectedly large. The experimental investigation of the correlation function in these kinematic regions has twofold interest, from one side it can help to investigate some delicate properties of quark jets and from the other side can help to verify the physical suppositions of model, and to obtain more precise values of parameters.

There are, of course, several obvious defects of the model, which were discussed by various people begun with authors of model, Field and Feynman. The detailed criticism of model can be found in work [3]. There are, also, many attempts to improve the model for the more precise description of the experimental data (see, for instance, Refs. [3] and [9]).

However, despite on the available defects, the model is able to describe the experimental data in DIS region with good enough precision, as the numerous applications are shown. Therefore, we think that it can be used for the qualitative study of the two hadron correlation functions at moderate energies.

And last point, which we would like to discuss here, is the place of this work among others, devoted to the investigation of the DFF. At present, many questions connected with the hadron-hadron correlation become the objects of the experimental investigations in the high-energy lepton-lepton, lepton-hadron, lepton-nucleus, hadron-hadron, hadron-nucleus and nucleus-nucleus interactions. As a consequence are arisen theoretical works, which devoted to the explanation and description of the existing experimental data or to the proposals for future experiments. There is huge amount of such experimental and theoretical works. Naturally it is impossible to present them in one article. We present only two of them, because they reflect common approach to the investigation of DFF on phenomenological level. They are Refs. [10], and [11]. In these works authors had concentrated on the definition of the DFF and derived the DGLAP evolution equations for

<sup>2</sup> Following FF2 we will call the isospin and strangeness properties the "flavor" of the primary mesons.

<sup>3</sup> It is worth to mention, that the correlations between final hadrons in the framework of bootstrap model, based on the recursive principle, was discussed, for the first time, in Ref. [8]. We are grateful to Professor A.Krzywicki for the comment concerning this point.

the non-singlet quark DFF [10] and singlet quark and gluon DFF [11]. The more attention was paid to the derivation of DGLAP evolution equations and less to the derivation of proper initial conditions. About initial conditions used in [10], authors themselves note, that in numerical study of the nonsinglet quark DFF they used a simple ansatz for the initial condition as  $D_{NS}(z_1, z_2) = D(z_1) \times D(z_2)$ , which at best is just a guess and differs significantly from the inherent hadron correlations in a single jet. In the next work [11] the initial conditions for the DFF were extracted from JETSET at a scale  $Q_0^2 = 2\text{GeV}^2$ . Although both the SFF and DFF evolve rapidly with  $Q^2$ , their ratio, which frequently is used for the comparison with experimental data, has a very weak  $Q^2$  dependence. This is especially true for the ratio in the case of the gluon fragmentation function, which shows practically no change with  $Q^2$ . The results of evolution are strongly dependent, however, on the initial conditions and thus on the actual values of  $z_1$  and  $z_2$ . In this work we study the DIS at moderate energies and  $Q^2$ , i.e. we investigate initial conditions for DFF for different pairs of hadrons and different values of  $z_1$  and  $z_2$ . We show, that different choice of the types of hadrons and their fractional energies leads to the essentially different initial conditions for DFF.

The paper is organized as follows. In Section 2 we briefly discuss the structure of FFQJM and point out the way of the derivation of the formulae for the calculations of SFF and DFF, and present expression for correlation function. Section 3 presents Results and Discussion. Our Conclusions are presented in Section 4.

## II. THEORETICAL FRAMEWORK

Let us consider the semi-inclusive DIS process on proton, in which a two hadron system is observed in the final state:

$$e + p \rightarrow e' + h_1 + h_2 + X \quad . \quad (1)$$

We would like to study correlation between two hadrons in the framework of the FFQJM. Before we want briefly remind, that main ingredients of FFQJM are the probability function  $f(\eta) = 1 - a + 3a\eta^2$  and three free parameters: 1) the ratio  $\gamma_s/\gamma_u$ ; 2) the probability of pseudoscalar meson production  $\alpha_{ps}$  ( $\alpha_{ps} + \alpha_v = 1$ ) and 3) the mean transverse momentum of the primary mesons. As we already mentioned in Introduction, we do not consider third parameter and suppose that integration over it is performed. It is easily to see, that parameters from points 1 and 2 are correlated with parameter  $a$  entering in function  $f(\eta)$ . In FFQJM the ratio  $\gamma_s/\gamma_u$  is fixed on value 0.5, the choice of parameter  $\alpha_{ps}$  and correlated with it parameter  $a$  leads to the two sets of parameters (two versions of model) considered in FF2. They are: a) simple version, which takes into account the direct production of pseudoscalar mesons only, with corresponding set of parameters  $a = 0.88$ ,  $\alpha_{ps} = 1$ ,  $\alpha_v = 0$ ; and b)

basic version, which takes into account both the direct production of pseudoscalar mesons and their production from the decays of resonances, with set of parameters  $a = 0.77$ ,  $\alpha_{ps} = \alpha_v = 0.5$ .

Now we introduce the notion "rank" following FF2. Let us consider the "hierarchy" structure of the final mesons produced when a quark of type "a" fragments into hadrons. New quark pairs  $b\bar{b}$ ,  $c\bar{c}$ ,  $d\bar{d}$  etc., are produced and "primary" mesons are formed. The "primary" meson  $a\bar{b}$  that contains the original quark is said to have "rank" one, the "primary" meson  $b\bar{c}$  "rank" two, the "primary" meson  $c\bar{d}$  "rank" three, etc. Finally, some of the "primary" mesons decay and it is assigned, that all the decay products to have the "rank" of the parent. The order in "hierarchy" is not the same as order in momentum or rapidity.

As it was mentioned above we use for SFF and DFF formulae obtained in FF2. Unfortunately, corresponding expressions are very long and inconvenient for reading. We do not present these formulae here, and only remind the numbers of corresponding equations in FF2. For SFF it is used formula (2.57) from FF2. The structure of DFF is more complicate and consists of five terms: (i) the probability that the primary meson at  $z_1$  is of rank 1 and the primary meson at  $z_2$  is of rank 2, given by (2.43a); (ii) the probability that  $z_1$  is of rank one, but  $z_2$  is of rank higher than 2, given by (2.43b); (iii) the probability that the primary meson at  $z_1$  is not first in rank but higher, but the primary meson at  $z_2$  is directly of next rank to the one at  $z_1$ , given by (2.43c); (iv) neither the primary meson at  $z_1$  or  $z_2$  is first in rank, nor are they adjacent, given by (2.43d). As it is pointed out in FF2, the complete DFF for producing two hadrons of flavor  $h_1 = ab$  and  $h_2 = cd$  is given by symmetrizing (2.43a-d) with respect to  $z_1$  and  $z_2$  (see eq. (2.44) of FF2). As pointed out in FF2, inclusion in the consideration the mesons produced from the decays of resonances complicates the structure of equations (2.43a-d) and adds next term (v) corresponding to the contribution from the case where both the mesons at  $z_1$  and  $z_2$  came from the decay of the same resonance (type  $h_v$ ) given by formula (2.59) of FF2.

There are two sources of correlations in the discussed model. Naturally, there is the correlation among secondary particles that are the decay products of the same primary meson. In addition, however, the primary mesons are not formed at random in  $z$ . Primary mesons adjacent in rank are correlated in both flavor and  $z$  since they each contain a quark (or antiquark) that came from the same  $q\bar{q}$  pair. The two primary mesons of adjacent rank tend to occur near each other in  $z$ . All flavor correlations in the quark jets occur between primary mesons of adjacent rank. The flavor of a meson of rank  $r + 2$  is independent of the flavor of the meson of rank  $r$ .

The observable which is widely used for the investigation of correlation between two hadrons produced in the any hard process is the two-body correlation function. It is a function of single and double hadron multi-

plicities  $\rho_1^h(z)$  and  $\rho_2^{h_1 h_2}(z_1, z_2)$ , respectively. In our case  $h_i$  denote the hadrons,  $i = 1, 2$ ;  $z_i$  denote the fraction of the virtual photon energy  $\nu$  carried by  $i$ -th hadron,  $z_i = E_i/\nu$  where  $E_i$  is the energy of the  $i$ -th hadron. In this work we will use, as observable, the "normalized" two body correlation function (see, for instance, FF2 and references therein)

$$R_{cor} = R^{h_1 h_2}(z_1, z_2) = \frac{\rho_2^{h_1 h_2}(z_1, z_2)}{\rho_1^{h_1}(z_1) \rho_1^{h_2}(z_2)} - 1 . \quad (2)$$

In the quark-parton model the single hadron multiplicity is

$$\rho_1^h(z) = \frac{1}{\sigma} \frac{d\sigma^h}{dz} = \frac{\sum_i e_i^2 q_i(x_{Bj}) D_i^h(z)}{\sum_i e_i^2 q_i(x_{Bj})} , \quad (3)$$

where  $\sigma$  and  $\frac{d\sigma^h}{dz}$  are inclusive and semi-inclusive cross sections,  $e_i$  and  $q_i(x_{Bj})$  are the electric charge of the quark in units of the elementary charge and distribution function of the quark with flavor  $i$  ( $i = u, d, s$ ) in proton,  $x_{Bj}$  is the Bjorken variable  $x_{Bj} = Q^2/2M_p\nu$ , where  $Q^2$  is the photon virtuality and  $M_p$  is the proton mass.  $D_i^h(z)$  is the SFF for the production of hadron  $h$  by  $i$ -th quark,  $z$  denote the fraction of the virtual photon energy  $\nu$  carried by hadron.

$$\rho_2^{h_1 h_2}(z_1, z_2) = \frac{1}{\sigma} \frac{d^2\sigma^{h_1 h_2}}{dz_1 dz_2} = \frac{\sum_i e_i^2 q_i(x_{Bj}) D_i^{h_1 h_2}(z_1, z_2)}{\sum_i e_i^2 q_i(x_{Bj})} , \quad (4)$$

where  $\frac{d^2\sigma^{h_1 h_2}}{dz_1 dz_2}$  is the semi-inclusive cross section in which two hadrons in the final state are observed. The DFF  $D_i^{h_1 h_2}(z_1, z_2)$ , are the distributions of hadrons of flavors  $h_1$  and  $h_2$  from a quark  $i$ , which carry away the fractions  $z_1$  and  $z_2$  of its energy, respectively.

### III. RESULTS AND DISCUSSION

We perform calculations mainly for basic version of the model. In this case it is necessary to take into account that a part of the final pseudoscalar mesons are arisen in result of decays of the other hadrons. We use contributions from 13 hadrons ( $K^0, \bar{K}^0, \rho^+, \rho^0, \rho^-, \eta, \eta', K^{*+}, K^{*0}, K^{*-}, \bar{K}^{*0}, \omega, \phi$ ). Further we will call them "resonances" although among them are long living hadrons also. Some of them, in result of decay, give contribution only in pion or kaon production, but there are resonances which decay into  $\pi K$  pairs. From eqs.(2)-(4) we are obtained the correlation functions  $R_{cor}$  for the pairs of mesons with different types and/or electric charges ( $\pi\pi, \pi K$  and  $KK$ ). For calculations were used expressions for SFF and DFF obtained in the framework of the FFQJM and the distribution functions of the quarks in the proton from [12]. For self-consistency of the consideration we use distribution functions of the quarks in the proton in the Leading Order (LO) approximation. For comparison with the basic version, we will present

results of calculations for simple version of model also.

In Fig.1 the correlation functions  $R_{cor}$  for the pairs

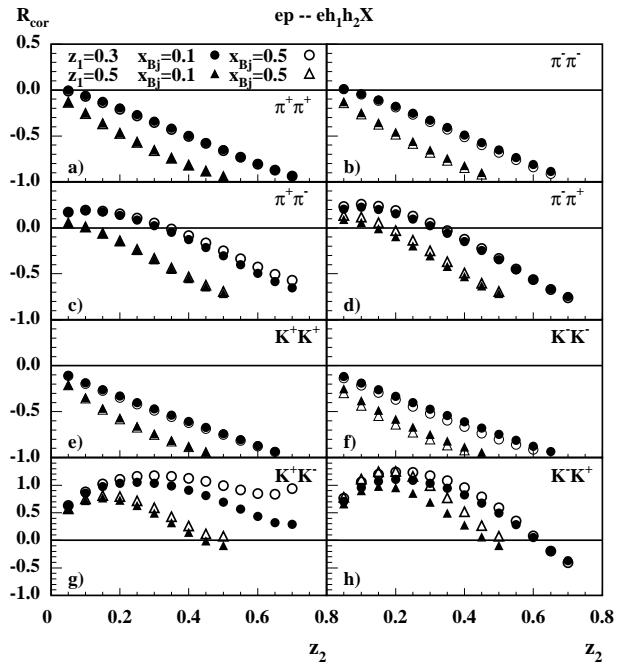


FIG. 1: Correlation functions  $R_{cor}$  for pairs of hadrons of the same type ( $\pi\pi$  and  $KK$ ), with identical and opposite electric charges as a function of  $z_2$  are presented. Calculations were performed for a fixed values of  $z_1$  equal to 0.3 (circles) and 0.5 (triangles). At each fixed value of  $z_1$  two curves with fixed values of  $x_{Bj}$  equal to 0.1 (filled symbols) and 0.5 (open symbols) are presented. The correlation functions were calculated in basic version of the FFQJM, taking into account hadrons produced both directly and from resonance decays. Only pairs of hadrons with different electric charges have positive correlation function.

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<sup>4</sup> We can not take for  $x_{Bj}$  too small values, because then become large the contributions of two jet events, which connected with the Pomeron exchange. In other words, it is need take into account that the part of its time in order of  $1/x_{Bj} M_p$  the virtual photon behaves as a  $q\bar{q}$  pair. If this time is essentially larger than the size of proton then instead of the photon with target will interact quark-antiquark pair. We can escape this situation imposing the restriction  $x_{Bj} \geq 0.1$ .

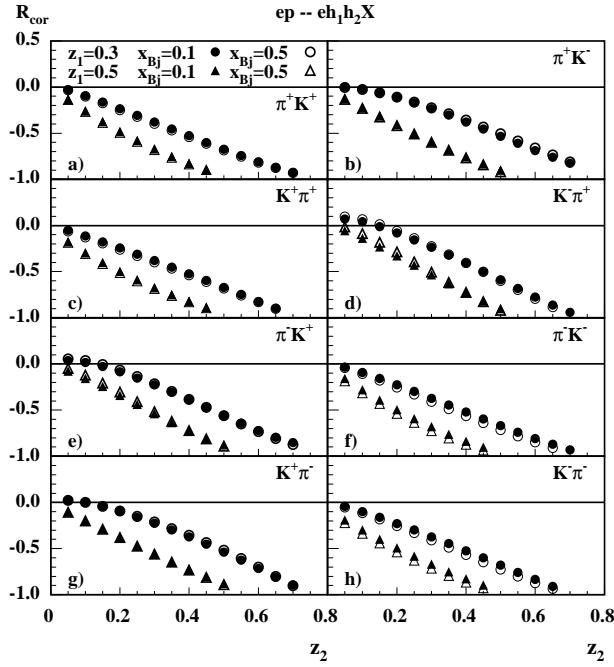


FIG. 2: The same as in Fig.1 for the pairs of hadrons of different types, having identical and opposite electric charges. Correlation functions are negative practically everywhere.

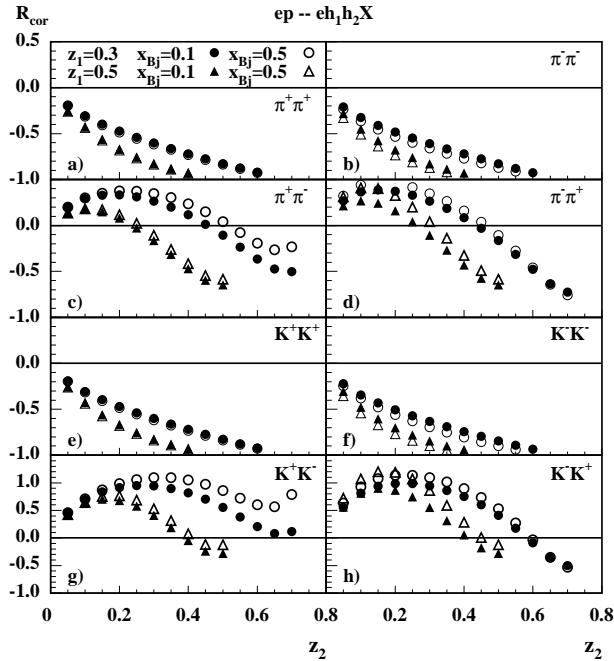


FIG. 3: The same as in Fig.1 for correlation functions calculated in the simple version of the FFQJM, taking into account only hadrons produced directly. Easily to see, that in comparison with basic version, absolute values of  $R_{cor}$  are here bigger.

functions for  $\pi^+\pi^-$  and  $\pi^-\pi^+$  pairs have similar shapes and become positive only at smaller value of  $z_1$  using

in calculations ( $z_1 = 0.3$ ). At bigger value of  $z_1$  using in calculations ( $z_1 = 0.5$ ) they are negative. In the cases of  $K^+K^-$  and  $K^-K^+$  pairs we see more strong positive correlation (pay attention on the scale of corresponding panels). Positive correlation persists practically over all range of the change of variables. Next peculiarity of the correlation functions for these pairs is the strong enough  $x_{Bj}$  - dependence. In case of  $K^+K^-$  more strong  $x_{Bj}$  - dependence takes place for  $z_1 = 0.3$ , while in case of  $K^-K^+$  it does for  $z_1 = 0.5$ . Pairs of hadrons with identical flavors (having the same type and identical electric charges) ( $\pi^+\pi^+$ ,  $\pi^-\pi^-$ ,  $K^+K^+$  and  $K^-K^-$ ) have correlation functions which are negative in all region of calculated  $z_2$ . They decrease (practically linearly) in all region of  $z_2$ . Later we will discuss the reasons of the different behavior of the pairs of hadrons of the same type, for cases, when they have opposite and identical electric charges.

In Fig.2 the correlation functions for the pairs of hadrons with different types ( $\pi K$  - pairs), with identical and opposite electric charges are presented. Notations are the same as in Fig.1. The correlation functions are calculated in basic version of the FFQJM. They are negative practically everywhere. Their behavior over variable  $z_2$  are close to the linear. We see only small differences in the shapes and values. The  $x_{Bj}$  - dependence is practically absent.

In Fig.3 the correlation functions  $R_{cor}$  for the pairs of hadrons of the same type ( $\pi\pi$  and  $KK$ ), with identical and opposite electric charges as a function of  $z_2$  are presented. Notations are the same as in Fig.1. The correlation functions are calculated in the simple version of the FFQJM, taking into account only hadrons produced directly. Easily to see, that in comparison with basic version, presented in Fig.1, absolute values of  $R_{cor}$  here bigger. The qualitative behavior of the correlation functions in basic and simple versions of model is very close. The positive correlation for  $\pi^+\pi^-$  and  $\pi^-\pi^+$  is more prominent in simple version, while for  $K^+K^-$  and  $K^-K^+$  pairs the situation is practically the same in both versions.

In Fig.4. the correlation function  $R_{cor}$  for  $\pi\pi$  pairs with opposite electric charges as a function of  $z_2$  is presented. Calculations were performed for a fixed values of  $z_1$  equal to 0.1, 0.3 and 0.5. The correlation function is calculated in the basic version of the FFQJM, taking into account hadrons produced both directly and from decays of resonances. It is presented the contributions of the different mechanisms in correlation function. The contribution of not adjacent in rank hadrons only (filled triangles); the sum of contributions of not adjacent and adjacent hadrons (open circles); and final result, the sum of three contributions: of not adjacent hadrons, adjacent hadrons and hadrons produced from the same resonance (filled circles).

In the Fig.5 the same as in Fig.4 for  $KK$  pairs is presented (pay attention on the scale).

Now let us discuss briefly the Figs.4 and 5. First ob-

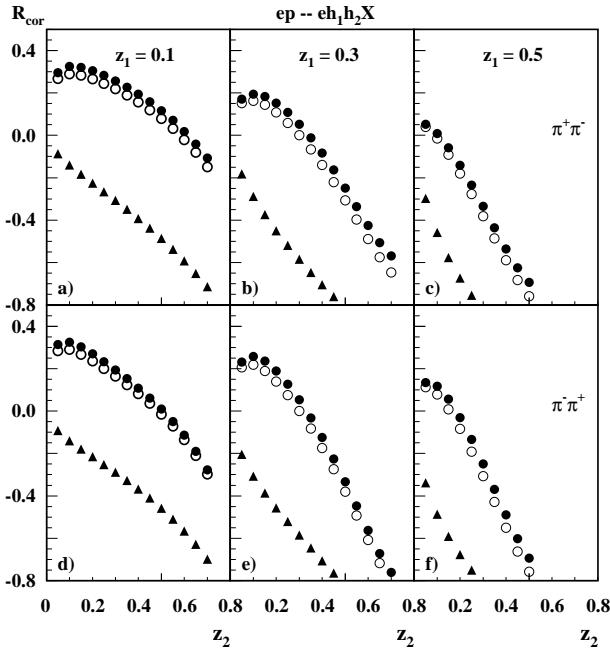


FIG. 4: Correlation functions  $R_{cor}$  for  $\pi\pi$  pairs with opposite electric charges as a functions of  $z_2$  are presented. Calculations were performed for a fixed values of  $z_1$  equal to 0.1, 0.3 and 0.5. The correlation functions were calculated in basic version of the FFQJM, taking into account hadrons produced both directly and from resonance decays. It is presented the contributions of the different mechanisms in correlation functions. The contribution of not adjacent hadrons only (filled triangles); the contribution of not adjacent and adjacent hadrons (open circles); the contribution of not adjacent and adjacent hadrons and hadrons produced from the same resonance (filled circles).

servation is, that taking into account only contributions from not adjacent mesons we obtain for pairs  $\pi^+\pi^-$ ,  $\pi^-\pi^+$ ,  $K^+K^-$  and  $K^-K^+$  the negative correlation functions which have the behavior very close to the cases for pairs of mesons of the different types, or for pairs of mesons of the same type and electric charges (identical flavors). The main source of the positive correlation is the possibility for the mesons entering in the pairs  $\pi^+\pi^-$ ,  $\pi^-\pi^+$ ,  $K^+K^-$  and  $K^-K^+$  to be produced in the adjacent rank. Second source of the positive correlation, the possibility for the mesons be produced from the decay of the same resonance is very small and can be neglected. From panels a, b, c of Fig.4 for  $\pi^+\pi^-$  pair, we see that while the contribution of not adjacent mesons is decreased with the increasing of  $z_2$ , the contribution of adjacent mesons is practically constant in all region of  $z_2$ . The similar behavior takes place for  $\pi^-\pi^+$  pair (see panels d, e, f of Fig.4). Situation is more complicated for the  $K^+K^-$  and  $K^-K^+$  pairs (see Fig.5). The behavior of the contributions from not adjacent mesons is like the  $\pi\pi$  case in Fig.4. The contribution of the adjacent mesons for  $KK$  pairs has more complicate behavior as a function of  $z_1$  and  $z_2$  and at some values of  $z_1$  and  $z_2$  can

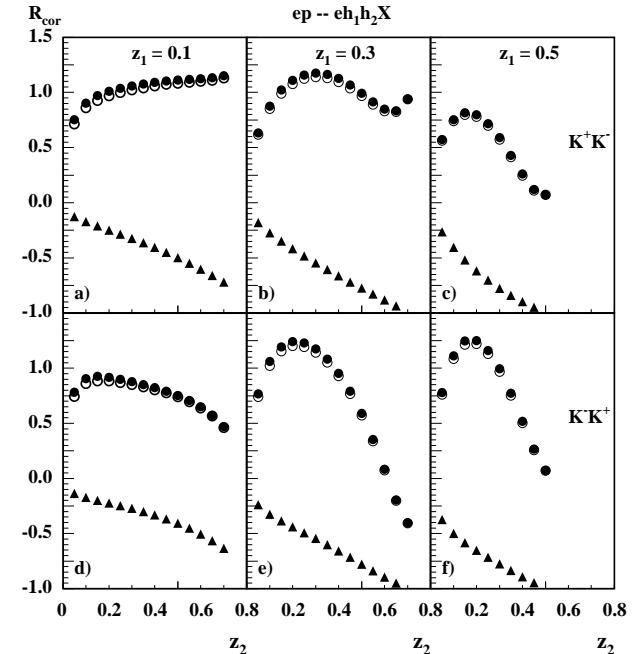


FIG. 5: The same as in Fig.4 for  $KK$  pairs. (Attention: other scale!)

be essentially larger than the contribution from not adjacent mesons. It is the source of the big and nontrivial positive correlations for the  $K^+K^-$  and  $K^-K^+$  pairs.

It is very important to verify, that results obtained above do not depend essentially from the choice of the parton distribution functions. For this goal we repeated all calculations with another set of parton distribution functions. Were used LO parton distribution functions from [13]. It was obtained, that difference between two results is smaller than 1 per cent.

We propose to measure the correlation functions for the  $K^+K^-$  and  $K^-K^+$  pairs experimentally. The experimental discovery of the fact of big positive correlation will means, that main mechanism of hadronization is the fragmentation of the initial quark via consecutive production of  $q\bar{q}$  pairs and their further combination into final mesons.

#### IV. CONCLUSIONS

In the absence of any dynamical correlations, the probability to observe in a single jet one hadron with fraction of energy  $z_1$  and second hadron with  $z_2$ , together with anything else, would be equal to the product of the probabilities to find hadrons with  $z_1$  and  $z_2$  in different jets and  $R_{cor}$ , as we defined it in eqs.(2)-(4), would be close to zero. Really, it is true only in case  $z_1 + z_2 \ll 1$ . When  $z_1 + z_2 \sim 1$  the correlation function for the pair of hadrons, which can not be produced adjacent in rank becomes negative, because of the necessity of the production of the additional

particles. Even in the case, when hadrons can be produced adjacent in rank part of energy is lost because the additional hadrons are produced in result of decays of resonances, which also leads to the reduction of correlations. We obtained such a behavior for all considered pairs of mesons, besides  $\pi^+\pi^-$ ,  $\pi^-\pi^+$ ,  $K^+K^-$  and  $K^-K^+$ . The prominent positive correlations for the  $K^+K^-$  and  $K^-K^+$  pairs in the framework of FFQJM were obtained. The experimental investigation of  $R_{cor}$  for these cases can help to verify the basic assumptions of the model and receive the valuable information about hadronization process.

The model does not include quantum effects. It has dealt solely with probabilities and not amplitudes. It is supposed that in DIS region the interference effects are small. If the interference effects, nevertheless, do not small, it will felt in selected pairs  $\pi^+\pi^-$ ,  $\pi^-\pi^+$  because of the interference between the amplitudes for the production of the hadrons  $h_1$  and  $h_2$  adjacent and not adjacent in rank. For the pairs  $K^+K^-$  and  $K^-K^+$  the interference will be practically impossible, because of the necessity of the production of the additional pair of  $s\bar{s}$ , and construction of specific final state.

The advantage of our approach to the derivation of SFF and DFF in comparison with, for instance, [10], and [11] is that we have, in the framework of FFQJM, the analytic formulae for the SFF and DFF, which are

obtained at moderate values of  $Q^2$ . They can serve as a initial state SFF and DFF for the QCD evolution. Moreover we can have separately the contributions in DFF from different mechanisms (production of hadrons adjacent and not adjacent in rank, production from the decay of the same resonance, etc.). It is very interesting to perform the QCD evolution for the different initial states and to trace the influence of the choice of the initial state, on the final state at high  $Q^2$ .

As we mentioned above, the main source of the positive correlation is the possibility for the mesons entering in the pairs  $\pi^+\pi^-$ ,  $\pi^-\pi^+$ ,  $K^+K^-$  and  $K^-K^+$  to be produced in the adjacent rank. This mechanism is not the peculiarity of the FFQJM and is available in all hadronization models of the fragmentation type (for instance [4] and [5]). It will be very interesting item to receive the correlation functions in other hadronization models.

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